

Linear Scaling Three Dimensional Fragment Method for Large Scale Electronic Structure Calculations

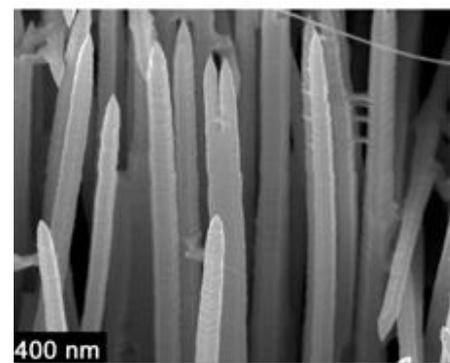
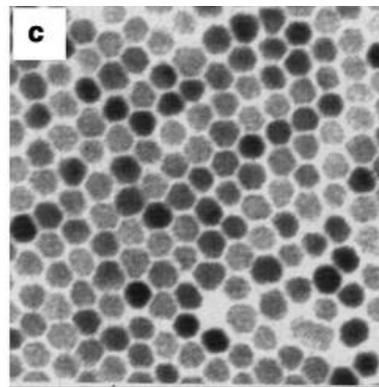
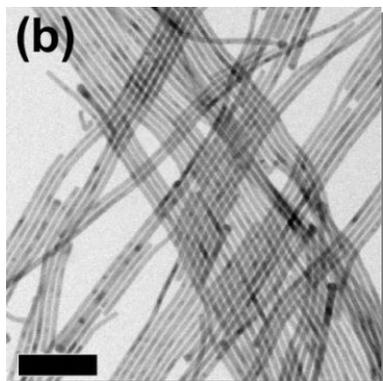
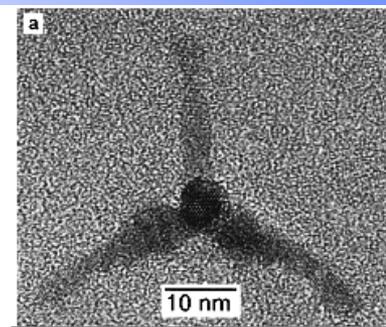
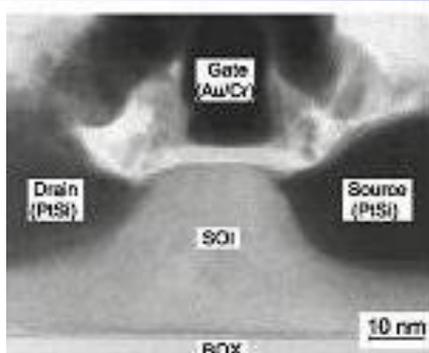
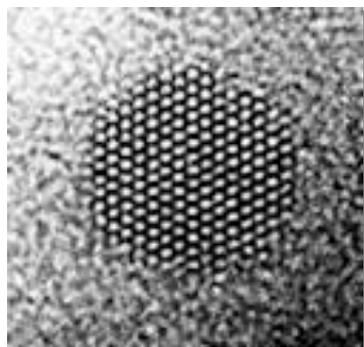
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Nanostructures have wide applications including: solar cells, biological tags, electronics devices



- ❖ Different electronic structures than bulk materials
- ❖ 1,000 ~ 100,000 atom systems are too large for direct $O(N^3)$ *ab initio* calculations
- ❖ $O(N)$ computational methods are required
- ❖ Parallel supercomputers critical for the solution of these systems

Why are quantum mechanical calculations so computationally expensive?

$$\left[-\frac{1}{2}\nabla^2 + V_{tot}(r)\right]\psi_i(r) = \varepsilon_i\psi_i(r)$$

- ❖ If the size of the system is N :
- ❖ N coefficients to describe one wavefunction $\psi_i(r)$.
- ❖ M ($i=1, M$) wavefunctions $\psi_i(r)$, M is proportional to N .
- ❖ Orthogonalization: $\int \psi_i(r)\psi_j^*(r)d^3r$, M^2 wavefunction pairs, each with N coefficients: $N*M^2$, i.e N^3 scaling.

The calculation of wavefunctions, and many of them, make the computation expensive, $O(N^3)$.

For large systems, need $O(N)$ method.

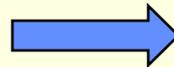
Previous Work on Linear Scaling DFT methods

- ❖ Three main approaches:
 - Localized orbital method
 - Truncated density matrix method
 - Divide-and-conquer method
- ❖ Some current methods include:
 - Parallel SIESTA (atomic orbitals, not for large parallelization)
 - Many quantum chemistry codes (truncated D-matrix, Gaussian basis, not for large parallelization)
 - ONETEP (M. Payne, PW to local orbitals, then truncated D-matrix)
 - CONQUEST (D. Bowler, UCL, localized orbital)
- ❖ Most of these use localized orbital or truncated-D matrix
- ❖ None of them scales to tens of thousands of processors

Linear Scaling 3 Dimensional Fragment method (LS3DF)

- ❖ A novel divide and conquer scheme with a new approach for patching the fragments together
- ❖ No spatial partition functions needed
- ❖ Uses overlapping positive and negative fragments
- ❖ New approach minimizes artificial boundary effects

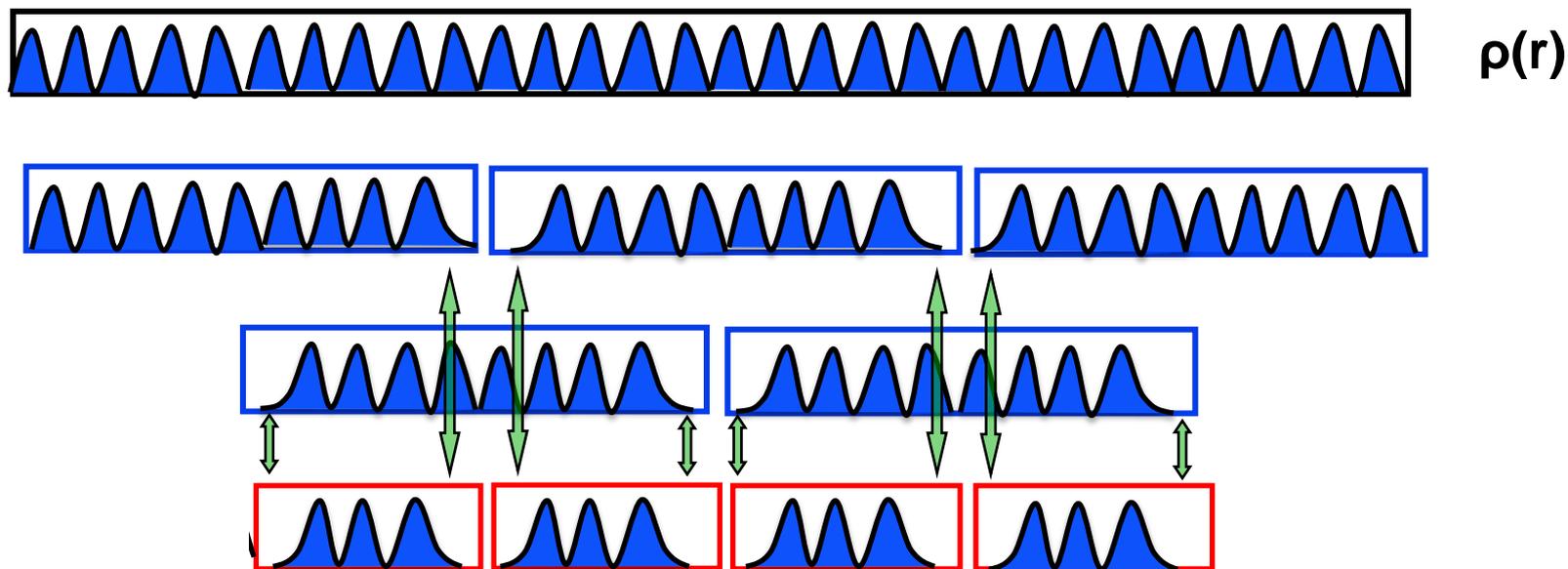
divide-and-conquer method



$O(N)$ scaling

Massively parallelizable

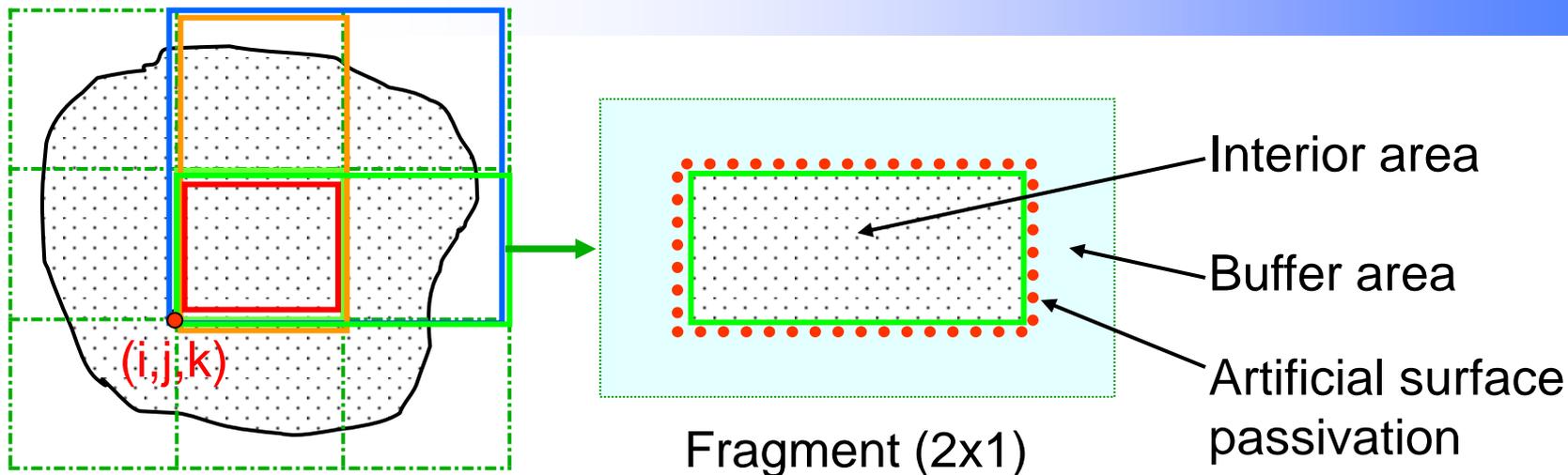
LS3DF: 1D Example



$$\text{Total} = \sum_F \{ \boxed{}_F - \boxed{}_F \}$$

Phys. Rev. B 77, 165113 (2008); J. Phys: Cond. Matt. 20, 294203 (2008)

Similar procedure extends to 2 and 3D

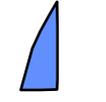
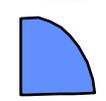
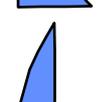
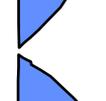
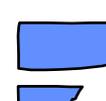
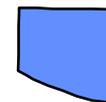
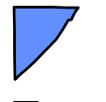
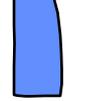
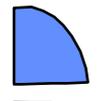
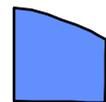
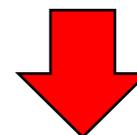
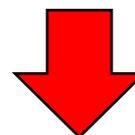
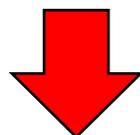
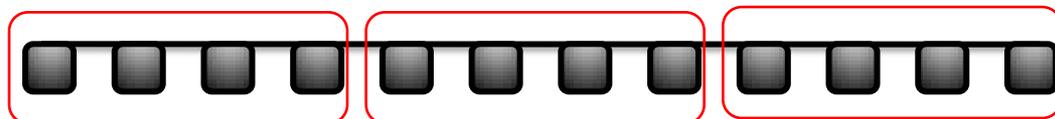
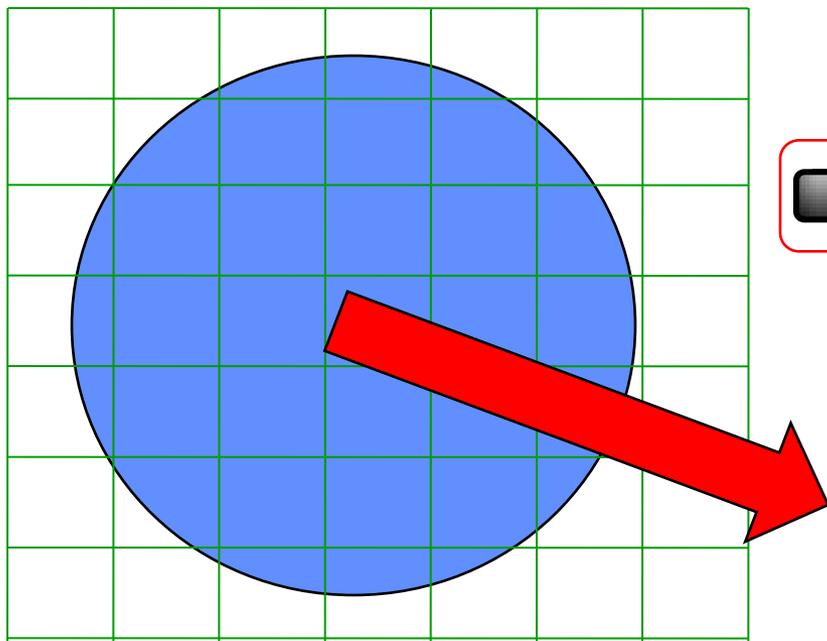


$$\text{Total} = \Sigma_F \left\{ \begin{array}{c} \text{Blue box} \\ \text{Orange box} \\ \text{Green box} \\ \text{Red box} \end{array} \right\}$$

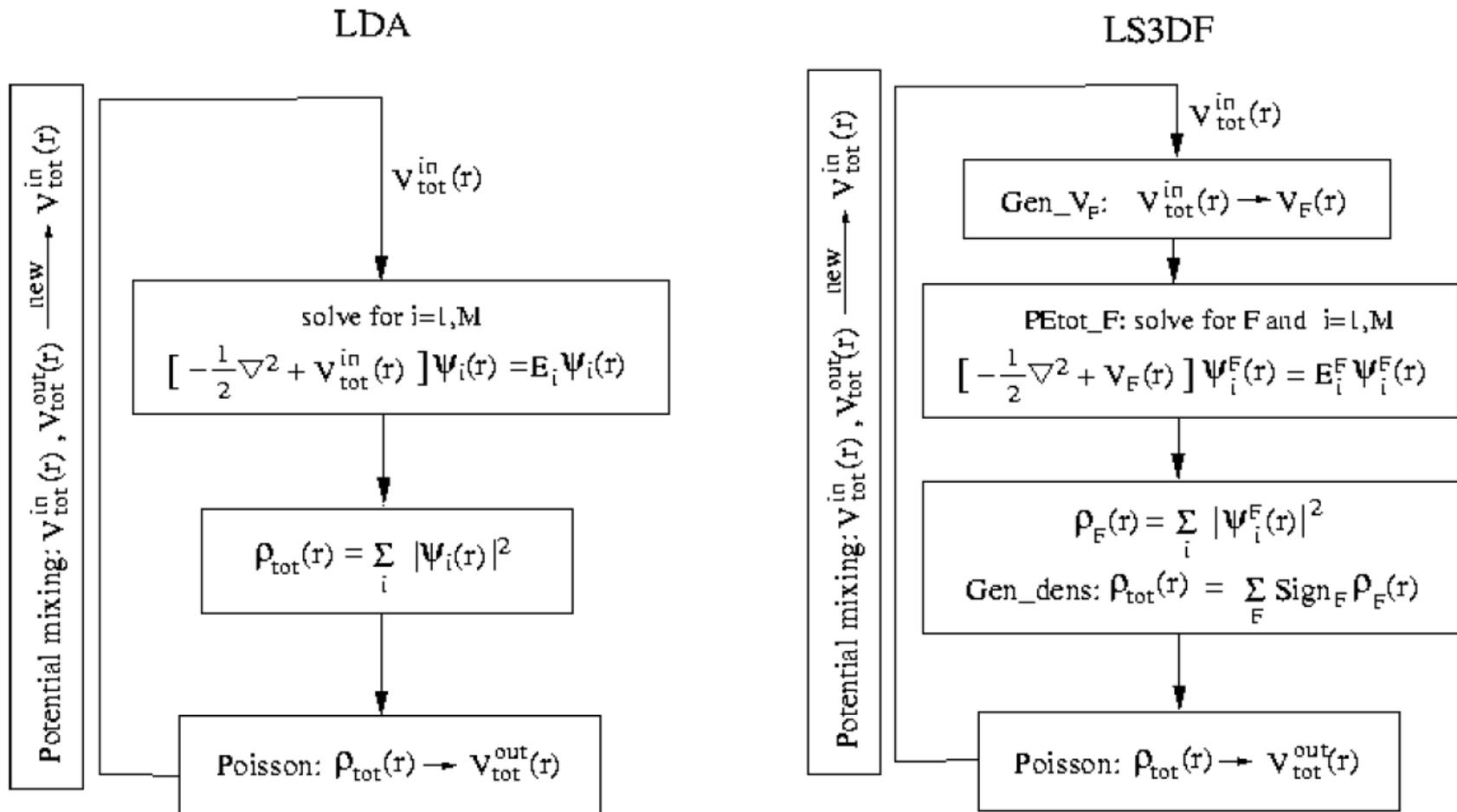
Boundary effects are (nearly) cancelled out between the fragments

$$\text{System} = \sum_{i,j,k} \left\{ F_{222} + F_{211} + F_{121} + F_{112} - F_{221} - F_{212} - F_{122} - F_{111} \right\}$$

Schematics for LS3DF calculation



Flow chart for LS3DF method



Based on the plane wave PEtot code: <http://hpcrd.lbl.gov/~linwang/PEtot/PEtot.html>

LS3DF Accuracy is determined by fragment size

- ❖ A comparison to direct LDA calculation, with an 8 atom 1x1x1 fragment size division:
 - The total energy error: 3 MeV/atom \sim 0.1 kcal/mol
 - Charge density difference: 0.2%
 - Better than other numerical uncertainties (e.g. PW cut off, pseudopotential)
- ❖ Atomic force difference: 10^{-5} a.u.
 - Smaller than the typical stopping criterion for atomic relaxation
- ❖ Other properties:
 - The dipole moment error: 1.3×10^{-3} Debye/atom, 5%
 - Smaller than other numerical errors

For most practical purposes, the LS3DF is the same as direct LDA

Some details on the LS3DF divide and conquer scheme

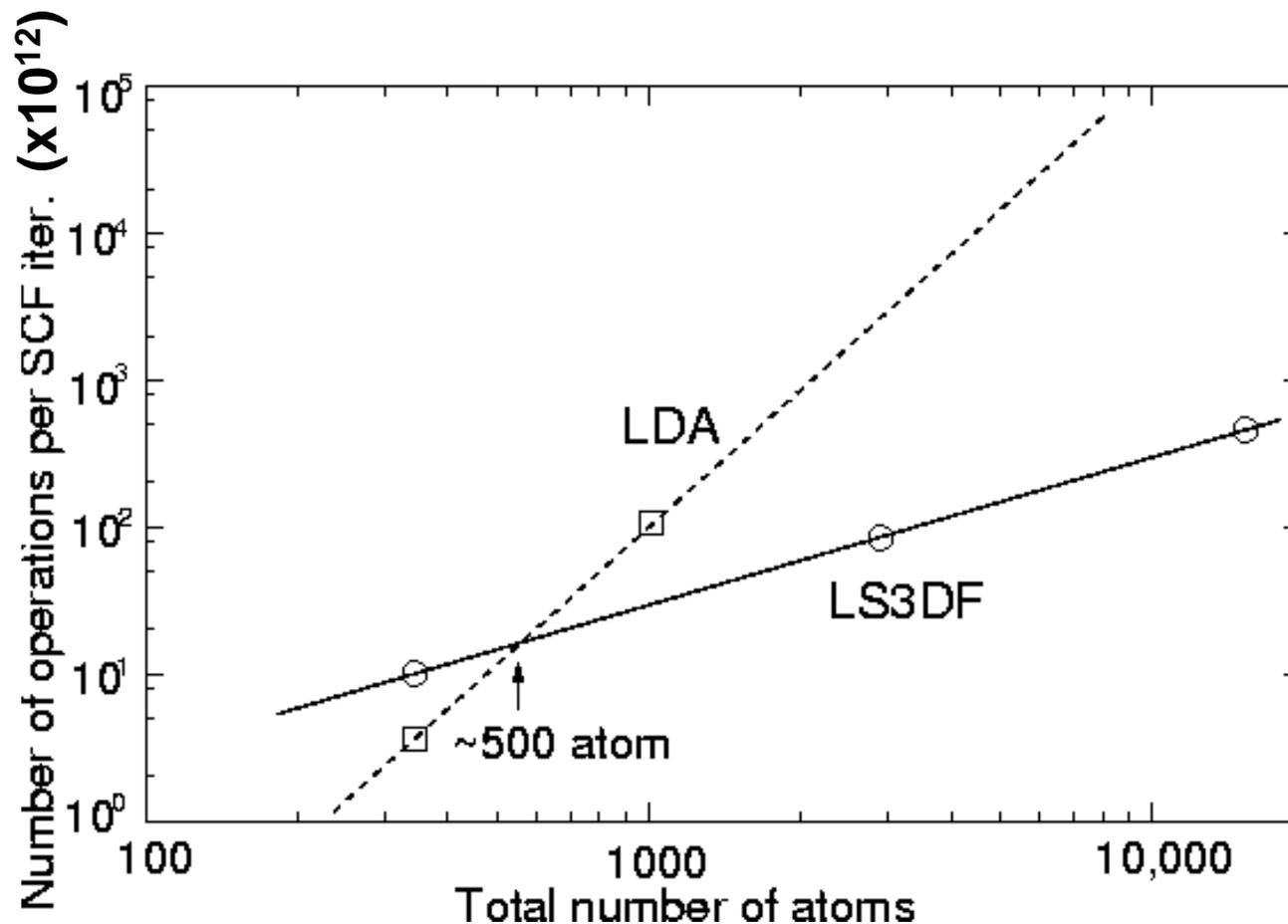
- ❖ Variational formalism, sound mathematics
- ❖ The division into fragments is done automatically, based on atom's spatial locations
- ❖ Typical large fragments (2x2x2) have ~100 atoms and the small fragments (1x1x1) have ~ 20 atoms
- ❖ Processors are divided into M groups, each with N_p processors.
 - N_p is usually set to 16 – 128 cores
 - M is between 100 and 10,000
- ❖ Each processor group is assigned N_f fragments, according to estimated computing times, load balance within 10%.
 - N_f is typically between 8 and 100

Overview of computational effort in LS3DF

- ❖ Most time consuming part of LS3DF calculation is for the fragment wavefunctions
 - Modified from the stand alone PEtot code
 - Uses planewave pseudopotential (like VASP, Qbox)
 - All-band algorithm takes advantage of BLAS3
- ❖ 2-level parallelization:
 - q-space (Fourier space)
 - band index (i in ψ_i)
- ❖ PEtot efficiency $> 50\%$ for large systems (e.g, more than 500 atoms), 30-40% for our fragments.

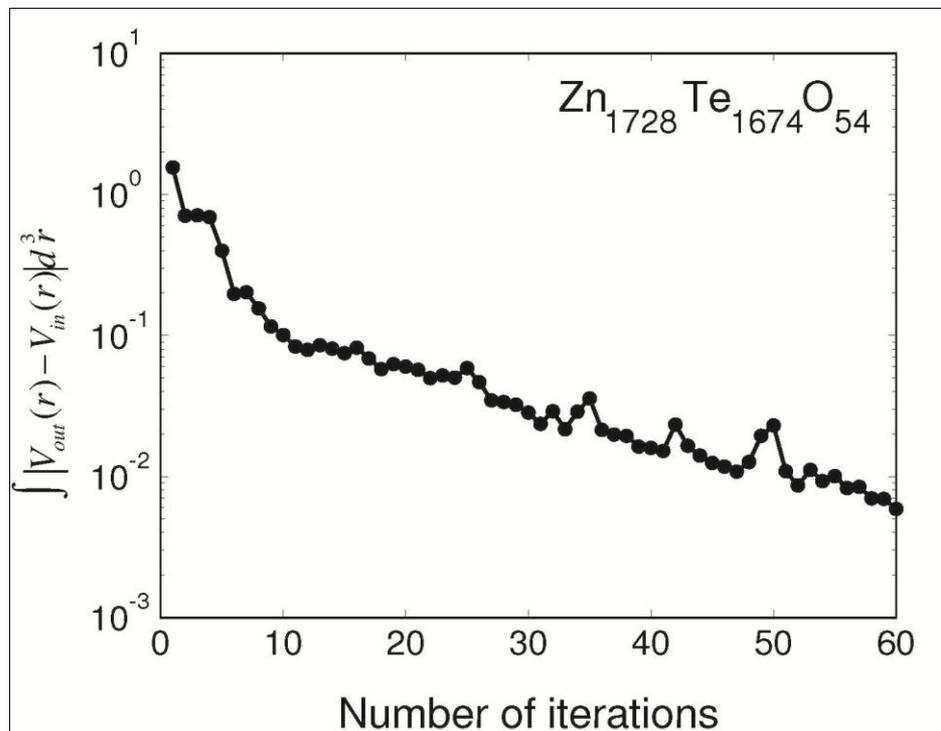
PEtot code: <http://hpcrd.lbl.gov/~linwang/PEtot/PEtot.html>

Operation counts

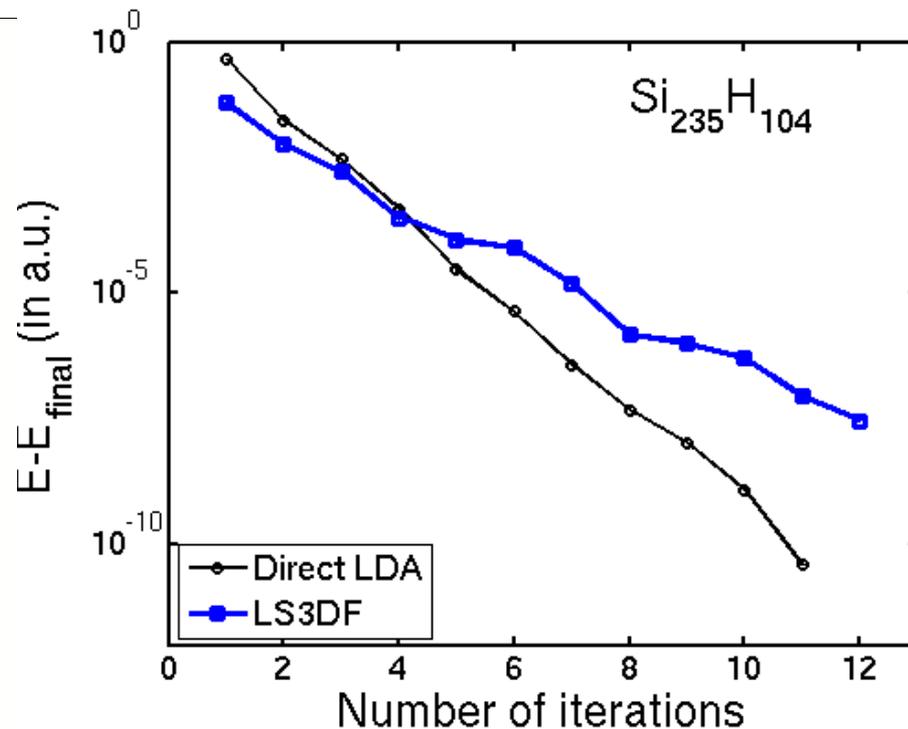


- ❖ Cross over with direct LDA method [PEtot] is 500 atoms.
- ❖ Similar to other $O(N)$ methods.

Selfconsistent convergence of LS3DF



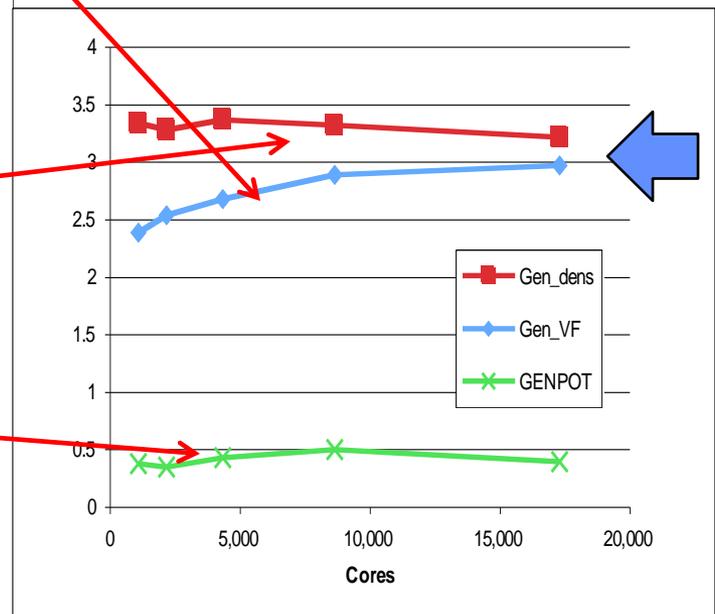
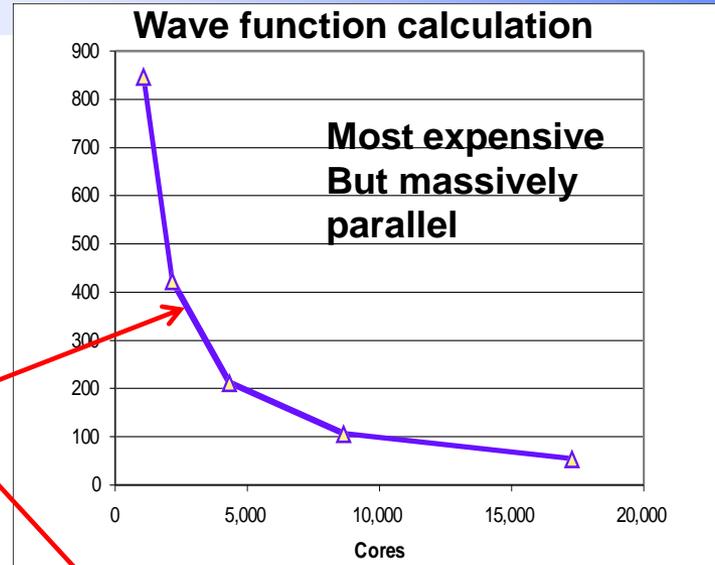
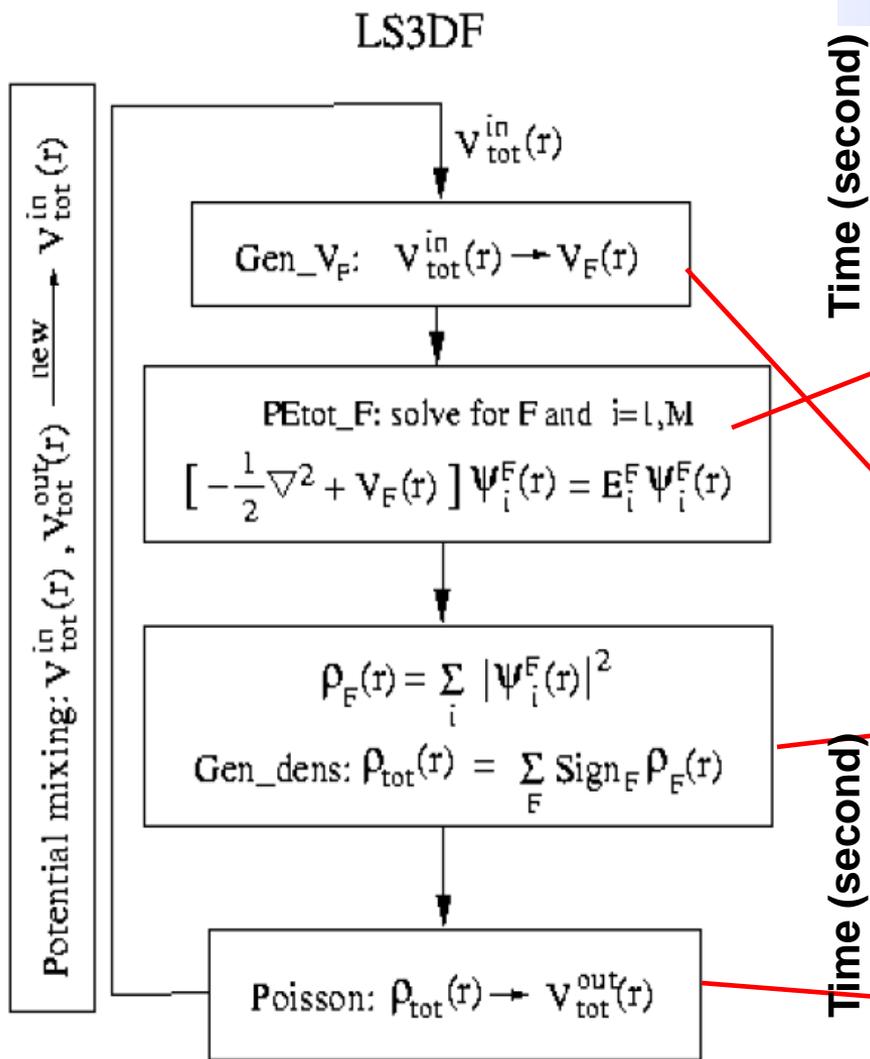
Measured by potential



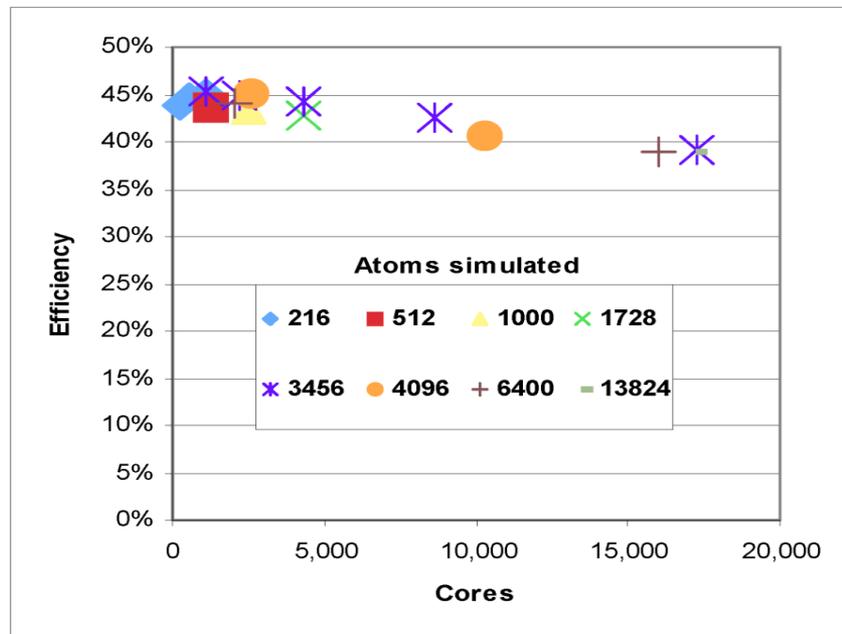
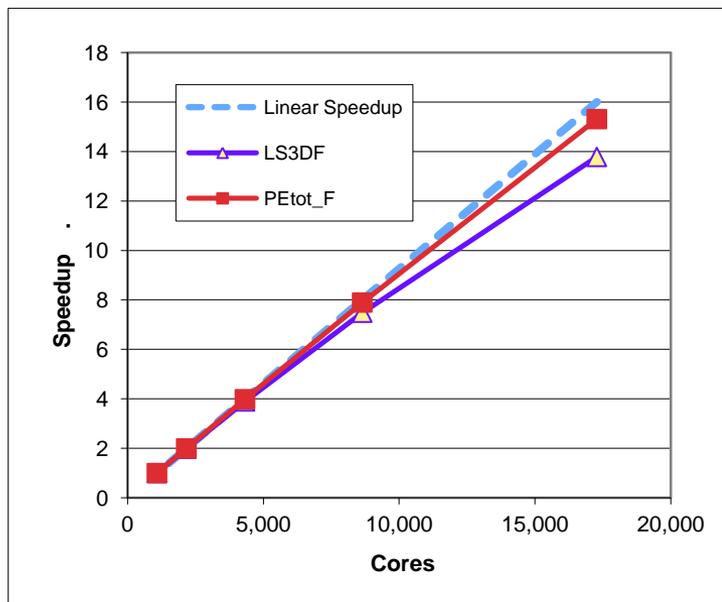
Measured by total energy

- ❖ SCF convergence of LS3DF is similar to direct LDA method
- ❖ It doesn't have the SCF problem some other $O(N)$ method have

The performance of LS3DF method (strong scaling, NERSC Franklin)

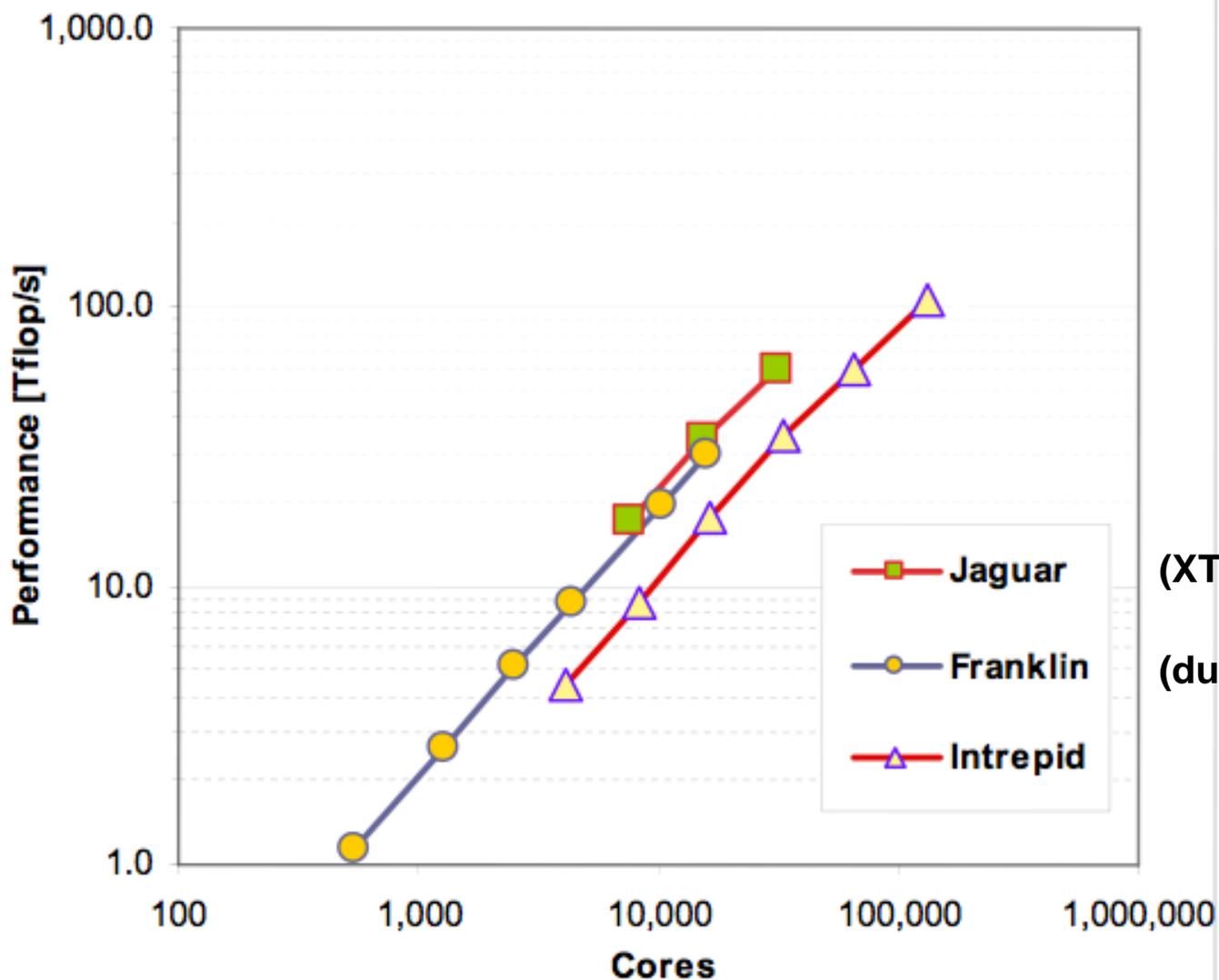


NERSC Franklin results (strong scaling)

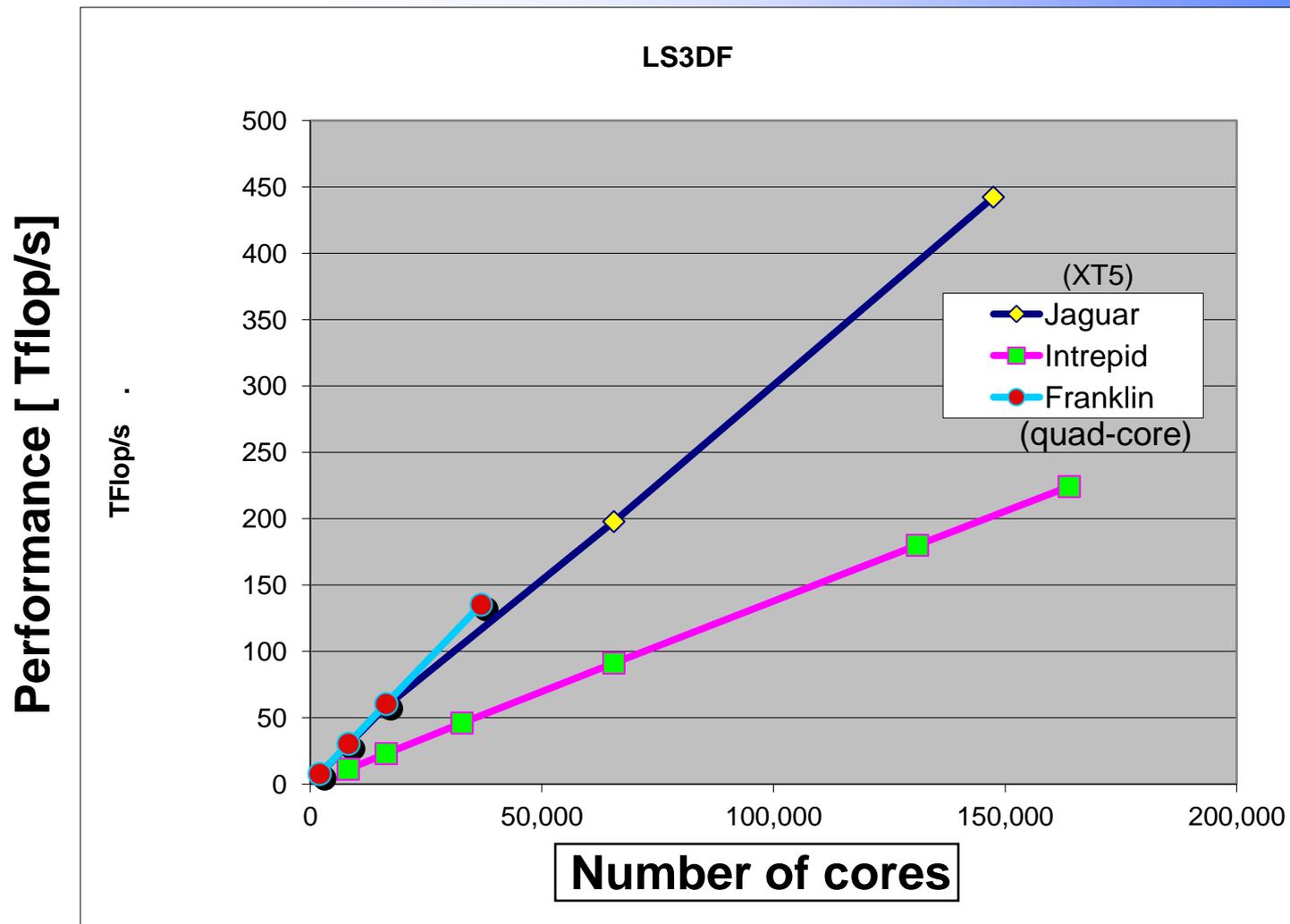


- ❖ 3456 atom system, 17280 cores:
 - one min. per SCF iteration, one hour for a converged result
- ❖ 13824 atom system, 17280 cores,
 - 3-4 min. per SCF iteration, 3 hours for a converged result
- ❖ LS3DF is 400 times faster than Petot on the 13824 atom system

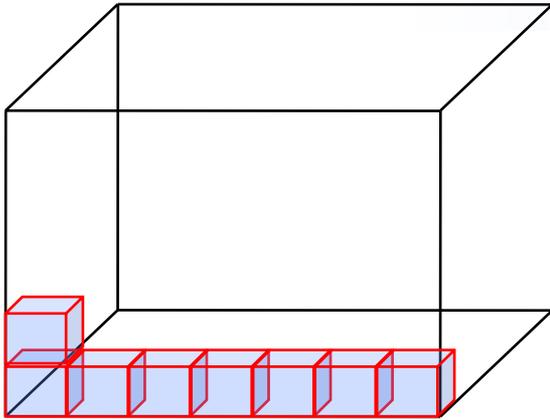
Near perfect speedup across a wide variety of systems (weak scaling)



ZnTeO alloy calculations (Ecut=60Ryd, with d states, up to 36864 atoms), weak scaling



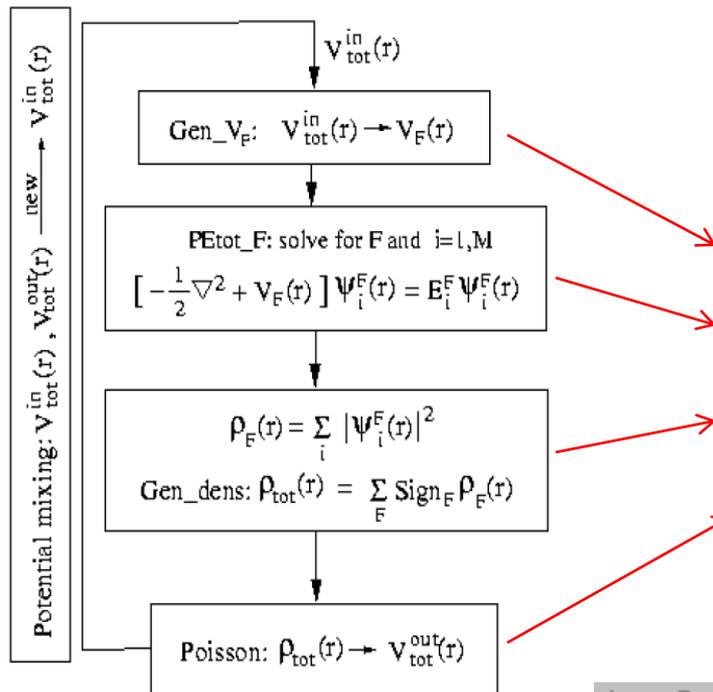
Node mapping and performance on BlueGene/P



Map all the groups into identical compact cubes, for good intra-group FFT communication, and inter-group load balance.

Time: 50% inside group FFT
50% inside group DGEM

LS3DF



Times on diff. part of the code (sec)

core	8,192	32,768	163,840
atom	512	2048	10,240
gen_VF	0.08	0.08	0.23
PEtot_F	69.30	68.81	69.87
gen_dens	0.08	0.14	0.37
Poisson	0.12	0.22	0.76

Perfect weak scaling

System Performance Summary



- ❖ 135 Tflops/s on 36,864 processors of the quad-core Cray XT4 Franklin at NERSC, 40% efficiency.

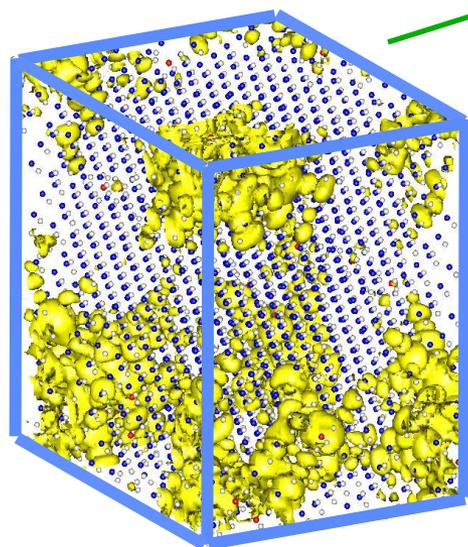
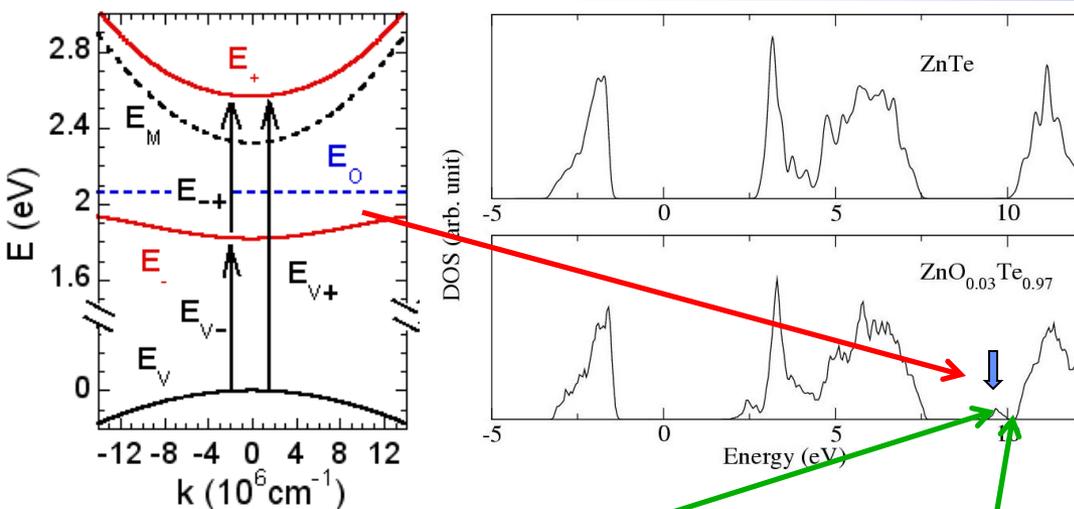


- ❖ 224 Tflops on 163,840 processors of the BlueGene/P Intrepid at ALCF, 40% efficiency.

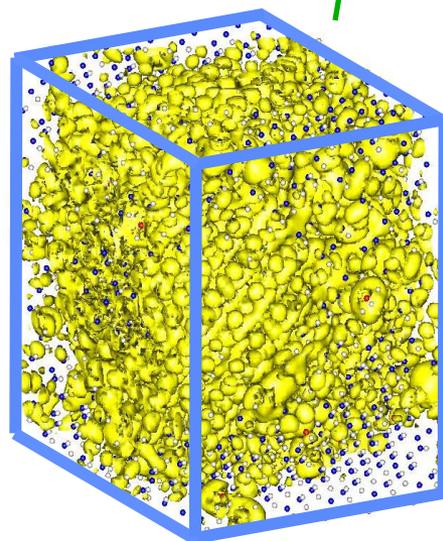


- ❖ 442 flops/s on 147,456 processors of the Cray XT5 Jaguar at NCCS, 33% efficiency.

Can one use an intermediate state to improve solar cell efficiency?



Highest O induced state

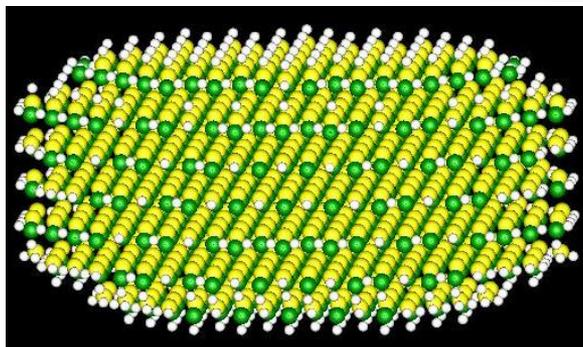


ZnTe bottom of cond. band state

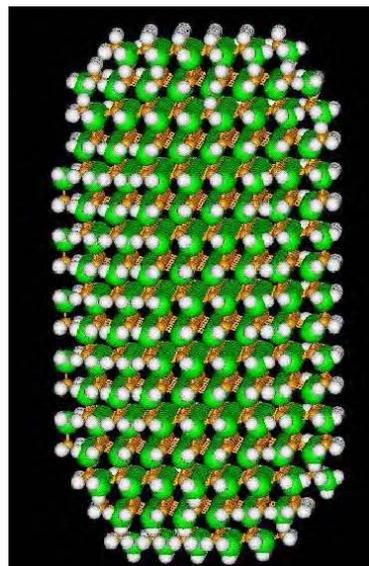
- ❖ Single band material theoretical PV efficiency is 30%
- ❖ With an intermediate state, the PV efficiency could be 60%
- ❖ One proposed material ZnTe:O
 - Is there really a gap?
 - Is there oscillator strength?
- ❖ LS3DF calculation for 3500 atom 3% O alloy [one hour on 17,000 cores of Franklin]
- ❖ Yes, there is a gap, and O induced states are very localized.

INCITE project, NERSC, NCCS.

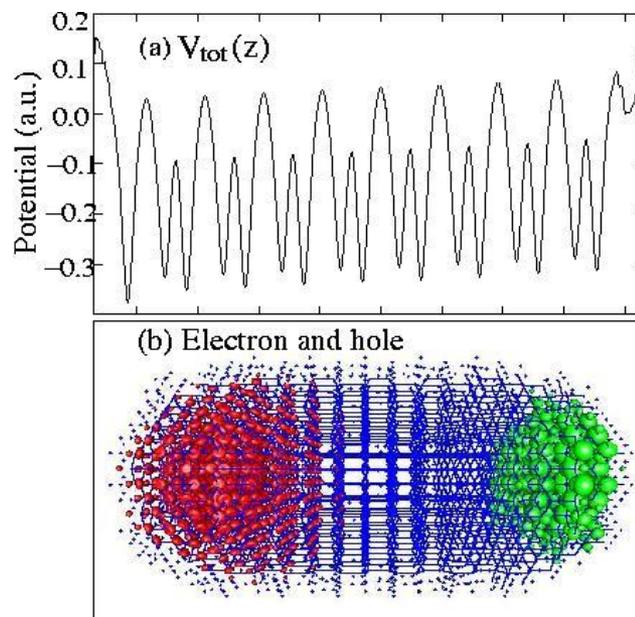
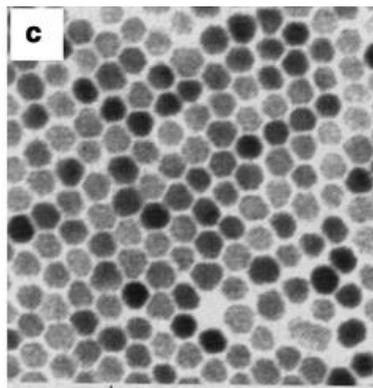
LS3DF computations yield dipole moments of nanorods and the effects on electrons



P = 30.3 Debye



P=73.3 Debye



$\text{Cd}_{714}\text{Se}_{724}$

WZ

- ❖ Equal volume nanorods can have different dipole moments
- ❖ The inequality comes from shape dependent self-screening
- ❖ Dipole moments depend on bulk and surface contributions
- ❖ Dipole moments can significantly change the electron and hole wave functions

INCITE project at NCCS and NERSC

Summary and Conclusions

- ❖ LS3DF scales linearly to over 160,000 processors. It reached 440 Tflop/s. It runs on different platforms without much retuning.
- ❖ For practical purposes, the numerical results are the same as a direct DFT based on an $O(N^3)$ algorithm, but at only $O(N)$ computational costs
- ❖ LS3DF can be used to compute electronic structures for $>10,000$ atom systems with total energy and forces in 1-2 hours. It can be 1000 times faster than $O(N^3)$ direct DFT calculations.
- ❖ Enables us to yield new scientific results predicting the efficiency of a proposed new solar cell material

Acknowledgements

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- ❖ **Innovative and Novel Computational Impact on Theory and Experiment (INCITE)**
- ❖ **SciDAC/PERI (Performance Engineering Research Institute)**
- ❖ **DOE/SC/Basic Energy Science (BES)**
DOE/SC/Advance Scientific Computing Research (ASCR)

LS3DF Team



Lin-Wang Wang



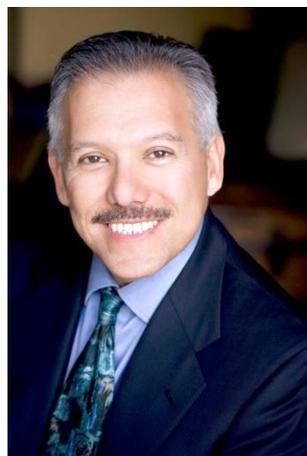
Zhengji Zhao



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Erich Strohmaier



David Bailey

Backup Slides

Variational formalism of LS3DF

Original DFT formula

$$E_{tot} = \sum_i \int \psi_i^*(r) \left[-\frac{1}{2} \nabla^2 \right] \psi_i(r) dr + \int V_{ion}(r) \rho_{tot}(r) dr + \frac{1}{2} \int \frac{\rho_{tot}(r) \rho_{tot}(r')}{|r-r'|} dr dr' + \int \varepsilon_{xc}(\rho_{tot}(r)) dr$$

LS3DF formula

$$E_{tot} = \sum_F \alpha_F \sum_i \int \psi_{F,i}^*(r) \left[-\frac{1}{2} \nabla^2 \right] \psi_{F,i}(r) dr + \sum_F \alpha_F \int \Delta V_F(r) \rho_F(r) dr + \int V_{ion}(r) \rho_{tot}(r) dr + \frac{1}{2} \int \frac{\rho_{tot}(r) \rho_{tot}(r')}{|r-r'|} dr dr' + \int \varepsilon_{xc}(\rho_{tot}(r)) dr$$

❖ The fragment wave function $\psi_{F,i}(r)$ is defined within each Ω_F

$$\int_{\Omega_F} \psi_{F,i}^*(r) \psi_{F,j}(r) dr = \delta_{i,j}. \quad \rho_F(r) = \sum_i |\psi_{F,i}(r)|^2 \quad \rho_{tot}(r) = \sum_F \alpha_F \rho_F(r),$$

Variational formalism of LS3DF

❖ Kohn-Sham equation of original DFT (N^3):

$$\left[-\frac{1}{2}\nabla^2 + V_{tot}(r)\right]\psi_i(r) = \varepsilon_i\psi_i(r)$$

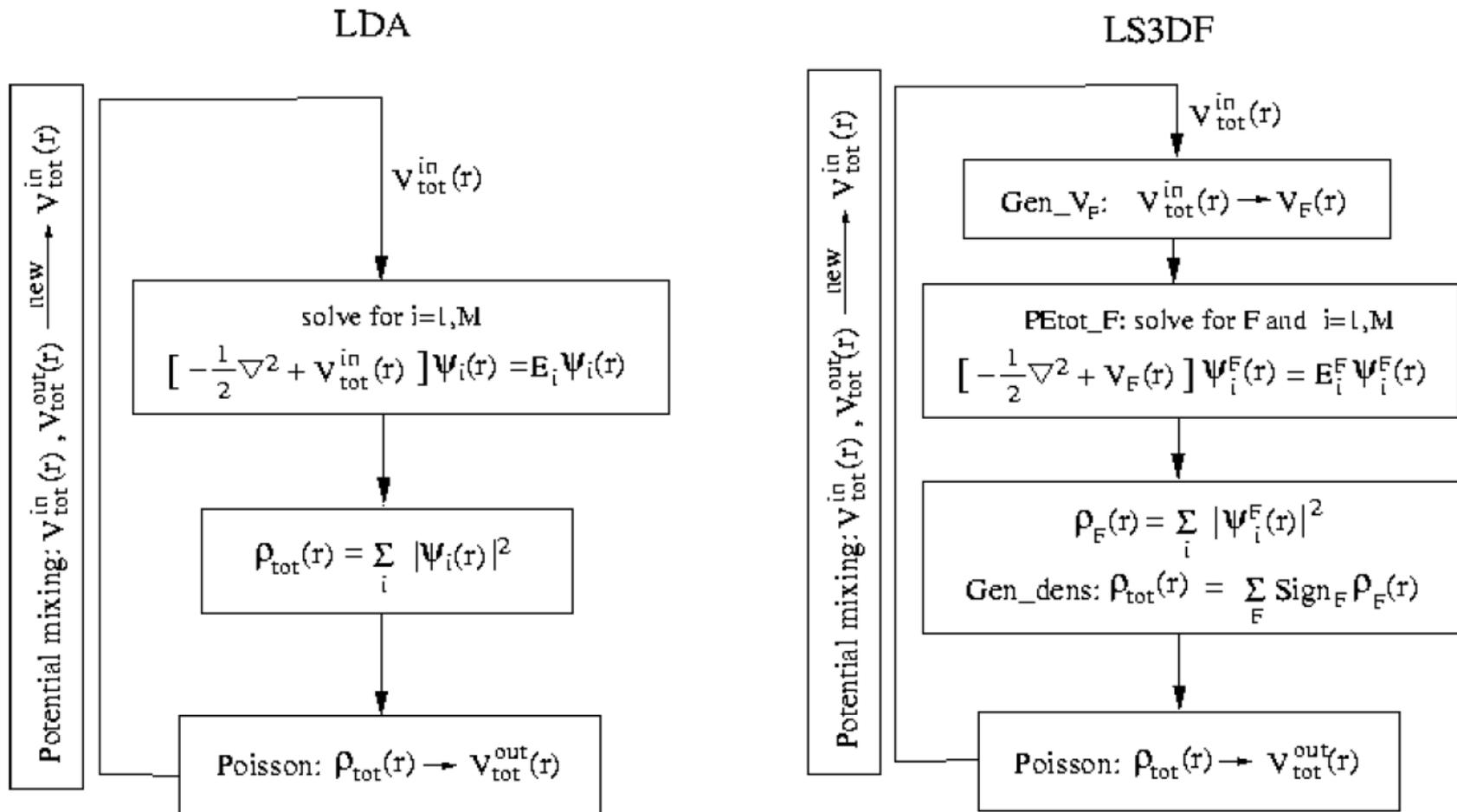
❖ Kohn-Sham equation of LS3DF :

$$\left[-\frac{1}{2}\nabla^2 + V_{tot}(r) + \Delta V_F(r)\right]\psi_{F,i}(r) = \varepsilon_{F,i}\psi_{F,i}(r) \quad \text{for } r \in \Omega_F$$

Where, $V_{tot}(r)$: usual LDA total potential calculated from $\rho_{tot}(r)$

$\Delta V_F(r)$: surface passivation potential

Flow chart for LS3DF method



Based on the plane wave PEtot code: <http://hpcrd.lbl.gov/~linwang/PEtot/PEtot.html>